Weighted averages

If all values in a set are equally weighted, the mean is simply the total sum of all the values divided by the number of values

$$\mu = \frac{\sum x}{N}$$

If the values are not weighted evenly, however, the total contribution of each value must be increased by an amount proportional to its weight. This total is then divided by the total of all the weights.

$$\mu = \frac{\sum (value \times weight)}{\sum weight}$$

Example #1: At a board meeting, participants can vote either yes (1) or no (0) on a proposal. Board members receive one vote, while members of the executive committee receive two votes. The vote will pass if it receives a simple majority ($\mu > 0.5$). If the executive committee votes 5 to 3 in favor and the rest of the board votes 7 to 4 against, then the average vote is given by

$$\mu = \frac{(\text{board member votes} \times 1) + (\text{executive committe votes} \times 2)}{\text{total votes}} = \frac{(5 \times 2) + (4 \times 1)}{16 + 11} = 0.52$$

and the motion passes.

Example #2: If homework counts for 30% of the grade, in a class, quizzes count for 20% and tests count for 50%, what is the overall grade of a student who scored 71%, 88% and 84% respectively?

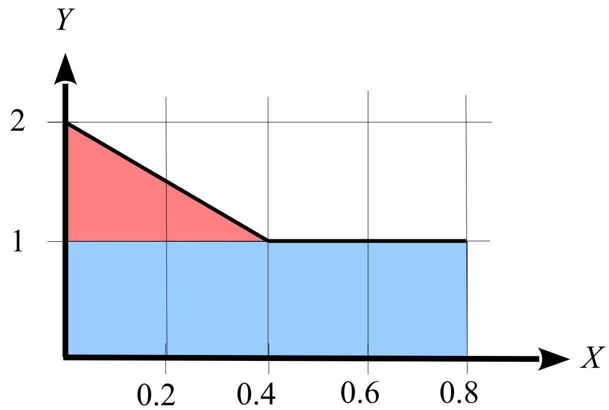
$$\text{overall grade} = \frac{(\text{homework grade} \times 30) + (\text{quiz grade} \times 20) + (\text{test grade} \times 50)}{100}$$

$$= \frac{(.71 \times 30) + (.88 \times 20) + (.84 \times 50)}{100} = 0.809 = 80.9\%$$

The student will receive a "B."

Example #3: For a probability density function, we can think of the mean as the point where the area under the graph would balance.

We can divide the probability distribution studied in class into two regions



The average (i.e. balancing point) of the blue region is simply 0.4 because it is symmetrical. Since the blue area comprises 4/5 of the total area, we assign it a weight of 0.8.

The average (i.e. balancing point) of the red region is $\frac{0.4}{3} \approx 0.1333$. (For an explanation of why this is so, click here.) Because its area comprises 1/5 of the total area, we assign it a weight of 0.2.

The weighted average of the graph is therefore

$$\mu = \frac{(\text{weight of blue region} \times \text{average of blue region}) + (\text{weight of red region} \times \text{average of red region})}{\text{total weight}}$$

$$= \frac{(0.8 \times 0.4) + (0.2 \times 0.1333)}{1} \quad \boxed{\approx 0.3467}$$

Notice that this answer is skewed to the right of the median (which was 0.31), but still to the left of center, as expected.